Linear Response: Rigorous Results and Applications

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Alessandra Faggionato Sapienza Università di Roma

Linear response, Nyquist relation and complex mobility matrix in periodically driven Markov processes

We perturbe a reversible stochastic process by a time-periodic external field introduced via a Girsanov-type transformation. We provide a martingale-based functional description of the linear response in the oscillatory steady state, and in particular of Nyquist relation. Considering the special case of diffusions on R^d with non-degenerate coefficients, we then analyze the standard complex mobility matrix and its extension to antisymmetric additive functionals. We discuss some analogous objects for random walks on a periodized random environment and show some preliminary result on the limit of the complex mobility matrix when the periodization length diverges. Based on two works in progress with P. Mathieu and M. Salvi, respectively.

Gary Froyland University of New South Wales

Optimal linear response for stochastic dynamical systems

We consider optimisation problems for discrete-time random dynamical systems, finding the perturbations that provoke maximal responses of statistical properties of the system. We treat systems whose transfer operator has an L^2 kernel, and consider the problems of finding (i) the infinitesimal perturbation maximising the expectation of a given observable and (ii) the infinitesimal perturbation maximising the spectral gap, and hence the exponential mixing rate of the system. Our perturbations are either (a) perturbations of the kernel or (b) perturbations of a deterministic map subjected to additive noise. We develop a general setting in which these optimisation problems have unique solution, construct explicit formulae for the unique optimal perturbations, and validate numerical methods for their approximation. We apply our results to a Pomeau-Manneville map and an interval-exchange map, both subjected to additive noise, to explicitly compute the perturbations provoking maximal responses.

John Harlim Penn State

Parameter Estimation of SDEs with Linear Response Theory

Parameter estimation is ubiquitous in the modeling of nature. In practice, the success of particular methods (either MLE or MCMC, Bayesian-based techniques) depends crucially on the choice of the loss function. In general, when the observable only depends on the parameters implicitly, an appropriate choice of the loss function is important for the identifiability of the parameters. In the context of Ito diffusion SDE's, a widely popular parameter estimation method is by fitting the empirical one- and two-point statistics, corresponding to equilibrium and dynamical properties of the system, respectively. In this talk, I will use linear response theory as a guideline for choosing the appropriate statistics to be fitted. I will discuss how we overcome several challenges in turning this idea into a numerically efficient and accurate algorithm. These include: (1) Solving the resulting dynamic-constrained least square problem with a convergence guaranteed polynomial surrogate model under the appropriate assumption. (2) Nonparametric estimation of Fluctuation-Dissipation-Theorem (FDT) response operator from time series with unknown density. (3) A generalization of this approach in the presence of modeling error. I will show numerical applications of coarse-graining of molecular dynamics.

Benoît Kloeckner Université Paris-Est Marne-la-Vallée

The linear Request problem

While the Linear Response problem asks whether the physical measure of a dynamical system is a differentiable function of the system and how the physical measure evolves under perturbation of the system, the Linear Request problem is the reciprocal question of controllability: given a perturbation of the physical measure, can one find a perturbation of the system that induces this particular evolution of the measure? This question was raised and studied recently by Stefano Galatolo and Mark Pollicott; the goal of the talk is to present a (very simple) general result, showing that when the physical measure and its perturbation are smooth the answer is positive without any dynamical assumption, but foremost to discuss the notion of perturbation of a dynamical system from a differential geometer's point of view.

Valerio Lucarini University of Reading

Predicting Climate Change through Linear Response Operators

Climate Models are key tools for predicting the future response of the climate system to a variety of natural and anthropogenic forcings. Here we show how to use statistical mechanics to construct operators able to flexibly predict climate change in a hierarchy of climate models, covering toy models, intermediate complexity models, and state-of-the-art general circulations models operationally used for climate projections. For the first time we prove the effectiveness of linear response theory for nonequilibrium systems in predicting future climate change associated with CO2 increase on a vast range of temporal scales, from inter-annual to centennial, and for very diverse climatic variables. We clarify the basic limitation of using standard forms of the fluctuation-dissipation theorem by elucidating the inability to reproduce non-trivial resonant responses associated with climate surprises. We investigate within a unified perspective the transient climate response and the equilibrium climate sensitivity, and assess the role of fast and slow processes. Linear response theory appears to have a very high predictive power regarding future changes of the state of the climate both for atmospheric and oceanic variables. We propose this methodology as an extremely efficient way to explore future climate change pathways and to test the safe operating space of our planet.

References:

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A. Gritsun V. Lucarini, Fluctuations, response, and resonances in a simple atmospheric model. Physica D 349, 62–76 (2017)

V. Lembo, V. Lucarini, F. Ragone, Beyond Forcing Scenarios: Predicting Climate Change through Response Operators in a Coupled General Circulation Model. Sci. Rep. 10, 8668 (2020)

M. Ghil, V. Lucarini, The Physics of Climate Variability and Climate, Rev. Modern Physics, 92, 035002 (2020)

Christian Maes KU Leuven

A trajectory-based approach to response theory

This talk is an introduction to various issues such as related to the mathematical and physical status of linear response, its extension to nonlinear response and the theory of response around nonequilibria. We emphasize the notion of dynamical ensemble and the crucial frenetic contribution in response and fluctuation theory. For reference: Christian Maes, Response theory: a trajectory-based approach. Frontiers in Physics, section Interdisciplinary Physics (2020).

Pierre Mathieu Aix-Marseille Université

Fluctuation-dissipation relations for reversible diffusions in a random environment

I will review recent results on fluctuation-dissipation relations for reversible diffusions in a random environment: Einstein and Nyquist relations as well as more general linear response formula. This review will be based on joint papers with Nina Gantert and Andrey Piatnitski and the recent PhD of Quentin Ghibaudo.
I will also mention open questions.

Grigoris Pavliotis Imperial College

Response theory and phase transitions for the thermodynamic limit of interacting identical systems

We study the response to perturbations in the thermodynamic limit of a network of coupled identical agents undergoing a stochastic evolution which, in general, describes non-equilibrium conditions. All systems are nudged towards the common centre of mass. We derive Kramers–Kronig relations and sum rules for the linear susceptibilities obtained through mean field Fokker–Planck equations and then propose corrections relevant for the macroscopic case, which incorporates in a selfconsistent way the effect of the mutual interaction between the systems. Such an interaction creates a memory effect. We derive conditions determining the occurrence of phase transitions specifically due to system-to-system interactions.
Such phase transitions exist in the thermodynamic limit and are associated with the divergence of the linear response but are not accompanied by the divergence in the integrated autocorrelation time for a suitably defined observable. We clarify that such endogenous phase transitions are fundamentally different from other pathologies in the linear response that can be framed in the context of critical transitions.
Finally, we show how our results can elucidate the properties of the Desai–Zwanzig model and of the Bonilla–Casado–Morillo model, which feature paradigmatic equilibrium and non-equilibrium phase transitions, respectively. This is joint work with Valerio Lucarini and Nicco Zagli.

Tomas Persson Lund University

Fractional response

I will talk about a joint work with Aspenberg, Baladi and Leppänen. If f_t is a family of piecewise expanding interval maps, and ϕ is an observable, not necessarily smooth, then (under some conditions) the response $t \mapsto \int \phi \, \mathrm{d}\mu_t$ has a fractional (Marchaud) derivative and the fractional derivative is related to a fractional susceptibility function. This work is an offspring of related results for the quadratic family by Baladi and Smania.

Mark Pollicott University of Warwick

Gibbs measures for hyperbolic attractors

We will describe constructions of Gibbs measures for hyperbolic attracting diffeomorphisms and flows by generalizing Sinai's original construction for SRB measures.

This is joint work with David Parmenter.

David Ruelle Université Paris Saclay

Guessing linear response for large chaotic systems

There are a number of naturally occurring large systems with apparently chaotic dynamics (turbulent hydrodynamical systems, large neural systems, etc.). There is a natural choice of physical steady states for such systems (SRB state) and a natural formal formal expression for the derivative of the SRB state with respect to the dynamics. We discuss the possible convergence of this formal expression and its interpretation as derivative.

Julien Sedro LPSM, CNRS

Quenched linear response for random hyperbolic dynamics

In this talk, I will explain how one may study the linear response problem for random compositions of (close-by) Anosov diffeomorphisms. This corresponds to establishing regularity, w.r.t parameters, of a family of equivariant measures, seen as living in the Gouëzel-Liverani scale of anisotropic spaces.
This is a joint work with Davor Dragicevic.

Daniel Smania University of Sao Paulo

Infinitesimal deformations of one-dimensional maps

Perhaps one of the main features of one-dimensional dynamics (either real or complex) is that the theory of deformations is rich.

By this we mean that the topological classes of such maps often are infinite dimensional manifolds with finite codimension and for smooth families of maps inside a given topological class the associated family of conjugacies also moves on a smooth way.

The derivative of this family of conjugacies with respect to the parameter is called an infinitesimal deformation.

There are various applications in the study of linear response problems and renormalisaton theory.

The theory of holomorphic motions and its uses in complex dynamics is one the most famous incarnations of this phenomena. Lyubich used infinitesimal deformation in a crucial way to study the quadratc-like maps and renormalization theory on this setting, as well as Avila, Lyubich and de Melo in the study of generic behaviour of quadratic unimodal maps.

For real maps on the interval this phenomena also occurs, but our current understanding is far behind the complex setting. We will discuss the recents developments obtained in joint works with several collaborators: Viviane Baladi, Amanda de Lima and Clodoaldo Ragazzo.

Fanni Sélley CNRS and Sorbonne Université

Linear Response for a Family of Self-Consistent Transfer Operators

A dynamical system is called self-consistent if the discrete-time dynamics is different in each time step according to some current statistics on the phase space. In this talk we study the special case of mean-field coupled smooth circle maps. Our main result is that for sufficiently weak coupling, the unique invariant density (the fixed point of the self-consistent transfer operator) is a C^1 function of the coupling strength. We present a linear response formula for its derivative reminiscent of the similar formula for families of C^2 circle maps. This is a joint work with Matteo Tanzi (Courant Institute).

Tatjana Tchumatchenko Max Planck Institute

Dynamical response functions in neuroscience

To understand how neurons function, it is essential to understand how they respond to changes in their environment. What are the changes in the neuronal environment that we should care about from the functional standpoint? It is crucial to understand how neurons encode small or large changes in their inputs because changes in neuronal spiking activity correlate with behavioral changes. Similarly, neurons stay alive by regulating their protein compositions to a particular dynamical set-point, and any deviation can lead to learning or cognitive deficits. Experimental approaches are needed to understand these questions because they can measure neuronal responses to targeted interventions. However, they are only part of the answer. Theoretical models are needed to make sense of the existing data and to make new, experimentally testable predictions. Key components of many mathematical models in computational neuroscience are linear and non-linear response functions. In this talk, I will present our recent work on two types of linear and non-linear response functions and outline how to obtain them.

Polina Vytnova University of Warwick

Computing the leading eigenvalue of the transfer operator: going beyond periodic points

Back in 2016, in a joint work with M. Pollicott, we gave explicit formulae, and computed
the first and second derivatives of the unique invariant measure of a one-parameter family of expanding maps with respect to a parameter. This method was based on a periodic point method for computing the leading eigenvalue of the transfer operator.

In the talk, I will present an efficient and effective alternative approach to computing
rigorous bounds on the leading eigenvalue of the transfer operator for an iterated function scheme.

Caroline Wormell LPSM, CNRS

Linear response in high-dimensional globally coupled systems

The response of long-term averages of observables of chaotic systems to small, time-invariant dynamical perturbations can often be predicted to first order using linear response theory (LRT), but some very basic chaotic systems are known to have a non-differentiable response. However, complex dissipative chaotic systems' macroscopic observables are widely assumed to have a linear response, but the mechanism for this is not well-understood.

We present a comprehensive picture for the linear response of macroscopic observables in high-dimensional weakly coupled deterministic dynamical systems, where the weak coupling is via a mean field and the microscopic subsystems may or may not obey LRT. Through a stochastic reduction to mean-field dynamics, we provide conditions for linear response theory to hold both in large, finite-dimensional systems and in the thermodynamic limit. In particular, we demonstrate that in systems of (even quite small) finite size, linear response is induced by self-generated noise.

Conversely, we will present an example in the thermodynamic limit where the macroscopic observable does not satisfy LRT, despite all microscopic subsystems satisfying LRT when uncoupled. This latter example is associated with emergent non-trivial dynamics of the macroscopic observable.

This is joint work with Georg Gottwald.
